Phase And Amplitude Balance: Key To Image Rejection Mixers

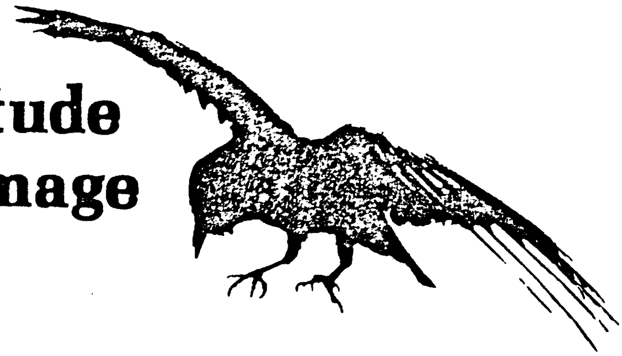


Image rejection mixers are ideally suited for broad band receivers where image rejection, spurious suppression and low noise figures are essential. Here is how they work.

ONE of the most important factors in the design of modern ECM receivers is being able to differentiate between the real and image signals. These image signals, Fig. 1, are created by the presence of unwanted rf signals at a given moment of search that are close enough to the LO frequency to be within the i-f bandwidth of the receiver. This situation is particularly true in an ECM environment, where there exists a multiplicity of radar signals, many of which can be confused with the desired signal sought. It is absolutely essential that the desired radar or transmitter signal be isolated from the image signal so direction finding and frequency counting techniques can be applied.

The simple solution of putting a bandpass filter at the rf port of the receiver, such as is done in communications systems, will just not work in an ECM environment. ECM bandwidth requirements are simply too broad, covering octave and even multi-octave bands. These requirements originate because hostile radar frequencies are seldom known exactly except within a given band. In addition, the radar can also be frequency agile further complicating the signal location problem.

In some ECM systems, electronic controlled preselectors in conjunction with rf amplifiers to compensate for their loss, are used to solve this problem. These techniques again are frequency, bandwidth and dynamic range limited and cannot be applied conveniently beyond L-band.

An image rejection mixer is an excellent means of meeting this problem. It consists basically of two conventional mixers, and 3 hybrids. The mixers may be either doubly balanced or singly balanced depending on design specifications. These components largely govern its band width limitations but octave and multi-octave bands can be designed.

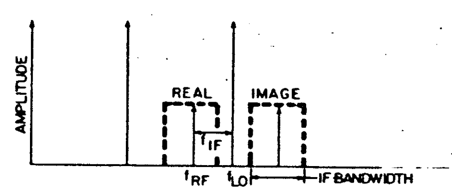
An image rejection mixer functions like a conventional mixer except it can differentiate whether the i-f output is due to a real rf or an image rf signal. This is achieved by means of phase cancellations which necessitates very rigid control of the phase and amplitude of the processed signal. Typically a phase balance of $\pm 5°$ and amplitude balance of ± 0.5 dB across an octave band is required to provide about 20dB of image rejection.

Since some phase and amplitude errors could occur at the interfaces between the hybrids and the mixer, the connection of individual coaxial components has not proven a very reliable technique in building image rejection mixers. Stripline and microstrip techniques are considered a better alternative depending on frequency and size constraints.

How does it work

The operation of an image rejection mixer can be explained by following the voltage vectors shown in Fig. 2. Assume circuit losses do not exist.

The model in Fig. 2 applies basically to a single balanced rejection mixer but the points apply to double balanced types as well. As may be observed, the output of the mixers at ports c and c′ provide voltage vectors $AV_1V_2/2$ and $BV_1V_2/2$ at frequency $f_{lo} - f_{rf}$ assuming the conversion loss of the two mixers is different. These voltage vectors ideally add at the real i-f output port and subtract at the image i-f output port. If $A \neq B$ or if these vectors are not in quadrature, a residual voltage exists at the image port. It is, therefore, very critical in a practical design that the hybrids are balanced in amplitude and phase so at the real port, the



1. Real rf output for an image rejection mixer occurs when $f_{LO} > f_{rf}$ and image i-f output occurs when $f_{rf} > f_{LO}$. This is arbitrary design decision and could be opposite.

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i-f output is $\frac{AV_1V_2}{2} + \frac{BV_1V_2}{2}$. If A = B, the real output signal is then equal to AV_1V_2. Of course this cannot be achieved in a practical design. For those interested in a more detailed design a mathematical approach is offered in the appendix.

The important design parameters in a practical image rejection mixer are:

- Image rejection
- Spurious suppression and dynamic range
- LO-rf isolation
- Conversion loss
- Noise figure
- LO, rf and i-f port VSWR
- Reproducibility, size and weight

The following discussion on these parameters apply to single balanced image rejection mixers. A comparison of the advantages of double balanced image rejection mixers and the single balanced image rejection mixers is provided later in the article.

Image rejection, how much?

Image rejection is the ratio of the image signal to the real signal at the real i-f port. Typically up to X-band, about 20 dB of image rejection can be realized. If initially the phases of all signal vectors are assumed balanced but the amplitudes are not, Fig. 2, then the image rejection is:

$$20 \log_{10} \frac{A+B}{A-B}$$

and for 20 dB of image rejection

$$\frac{A+B}{A-B} = 10$$

Then:

$$A = \frac{11}{9} B = 1.22B$$

This means a total maximum amplitude imbalance of 1.7 dB can exist.

If on the other hand, the amplitudes of all the signal vectors around the circuit in Fig. 2 are balanced but phases are not, the resultant amplitude vector is AV_1V_2. Since A = B, let the residual amplitude caused by phase distortion be K. Then for 20 dB image rejection:

$$\frac{2A}{K} = 10$$

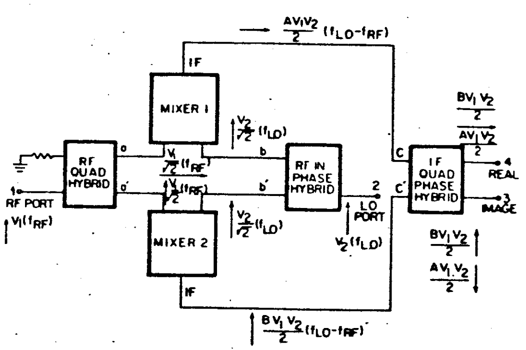
By the cosine law:

$$K^2 = A^2 + A^2 - 2A^2 \cos\theta$$
$$= 2A^2 [1 - \cos\theta]$$

Therefore $\cos\theta = 0.98$

$$= 11° 28'$$

Thus about 11° of total maximum phase imbalance in the voltage vectors at the input to the i.f. hybrid can exist. Since in practical designs, amplitude and phase imbalance co-exist, design goal limits must be less than these calculated values.



2. **Vectorial operation of a basic image rejection mixer shows ideally, complete cancellation at the i-f hybrid image port and phase reinforcement at the real port. In this case the real i-f output exists when f_{lo} is greater than f_{rf} but this is arbitrary.**

Table 1 — Test data of IM products for an S-band image rejection mixer

$mf_{lo} \pm nf_{rf}$

M \ N	1	2	3	4	5
1	0db* / 2.940 GHz	−40 / 1.470 GHz	−48 / 0.980 GHz	−80 / 0.735 GHz	>80 / 0.588 GHz
2	−20 / 5.940 GHz	−39 / 2.970 GHz	−56 / 1.980 GHz	>80 / 1.485 GHz	>80 / 1.188 GHz
3	−20 / 8.940 GHz	−34 / 4.470 GHz	−67 / 2.980 GHz	−78 / 2.235 GHz	>80 / 1.788 GHz
4	−29 / 11.940 GHz	−50 / 5.970 GHz	−58 / 3.980 GHz	−66 / 2.985 GHz	>−80 / 2.388 GHz
5	−29 / 14.940 GHz	−48 / 7.470 GHz	−54 / 4.980 GHz	−78 / 3.735 GHz	>80 / 2.988 GHz

* dB BELOW 60 MHz REAL IF OUTPUT.

Spurious suppression, a big problem

In the image rejection mixer, all intermodulation and cross modulation products of the rf and LO signals generated by the diodes must be controlled. These currents at frequency $mf_{LO} \pm nf_{rf}$ are generated by the diodes. For broad bandwidth or octave bandwidth image rejection mixers, the diode currents of particular importance exist within the i-f output bandwidth. The magnitude of the in-band spurious currents or voltages across the i-f load depends on the drive levels of the input LO signal and rf signal. For example, if the LO signal level is higher than the rf signal by 10 to 20 dB, spurious levels would be lower than if the LO signal level is only 3 to 6 dB above the rf signal. Table 1 shows typical spurious performance that would occur in an S-band image rejection mixer, with the LO power = 7.5 dBm and rf power = 10 dBm. The LO frequency is 3 GHz and the rf frequency is the bottom numbers shown in the table.

In image rejection mixers those spurious of importance are generally those due to 2×1, 2×2, 1×2, 3×2, 2×3 products. All other products within the band are generally 60 to 70 dB down. There are other products that could be only 20 dB below the signal level, but these exist at much higher frequencies.

For a single balanced image rejection mixer these spurious signals must be controlled by using variable bandpass filters or remotely switching bandpass filters in series with the input RF signal (Fig. 3). Unfortunately this adds additional insertion loss to the design of the receiver front end and increases the noise figure. A compromise is generally sought therefore between bandwidth, spurious suppression and noise figure.

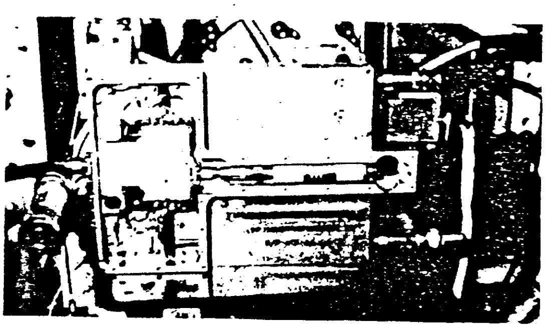
Cancel out 2×2 products

In-band spurious currents due to 2×2 products warrant the most concern in the design of image rejection mixers. (See Appendix). Cancellation of these currents is achieved in the single balanced mixers by properly biasing the diodes and keeping proper phase balance between rf signals injected into the diodes.

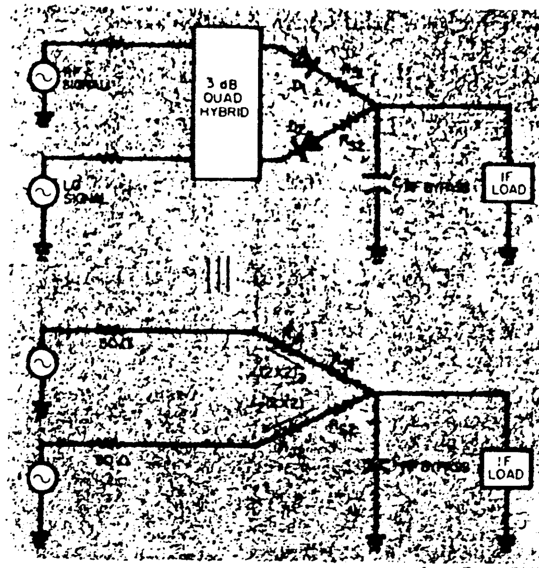
Cancellation of these 2 × 2 products depends on how well matched are the mixer diodes. In Fig. 4, R_{s1} and R_{s2} are the bulk resistances of the two diodes D1 and D2. R_{j1} and R_{j2} are the dynamic resistances of the diodes and are dependent on the dc bias level and LO signal level. (Assume that parasitics of the diodes are negligible. This is true at low frequency when beam lead diodes are used.) Currents, i_1 (2×2) and i_2 (2×2), exist in diodes D1 and D2 respectively. For infinite cancellation of any in-band 2×2 current flowing in the i-f load, junction "0" has to be at virtual ground potential. This can be achieved if $R_{j1} + R_{s1} = R_{j2} + R_{s2}$. Bulk resistance R_{s1} and R_{s2} vary in diodes from 5 to 15 ohms depending on the frequency of operation. No two diodes have equal R_s resistance even if the chips are selected from adjacent portions of a single wafer.

Matched diodes important

Manufacturers can supply diodes that are matched within certain specifications. By measuring the static I/V characteristics, select diode (balanced to within ±10 mV at 1 mA) can be designated from a wafer batch. To be certain to get truly matched diodes, it's important that they are matched within ±1 mV at 1 mA, 10 mA and 20 mA. This process however is time consuming and impractical for large scale production.



3. This complete r.f. front end uses switchable rf band pass filter for in band spurious level control. A low pass filter is also used in series with the microstrip image rejection mixer.



4. Equivalent circuit of balanced mixer shows presence of 2×2 currents in diodes D1 and D2.

R_{j1} and R_{j2} are dependent on the effective power level absorbed by the diodes. In practice this level in the two diodes is different due to parasitics and from the assembly of the diode chip on the alumina substrate. Also, the power level to the diodes are different due to the slight inherent imbalance of the 3 dB quad hybrid in the mixer. About ±0.2 dB amplitude imbalance can be expected. Since R_{j1} and R_{j2} are also dependent on the dc diode current, maximum cancellation of the 2×2 current in the i-f load can be achieved by adjusting the dc current levels so that the sum of $R_{j1} + R_{j2}$ is equal to $R_{j2} + R_{s2}$. In practice, 40 dB to 45 dB of 2×2 spurious signal cancellation is achievable in a single balanced mixer by this approach, over an octave bandwidth.

Sometimes it is desirable to equalize the 2×2 levels at the image and real i-f output coupler instead of biasing for maximum suppression at the real port. This is done in some ECM applications where the 2×2 product can be used for determining certain rf phase relationships. In this case the diodes must be rebiased in the mixer and in doing this, the 2×2 level will be increased by about 5 dB. See Fig. 5.

Conversion loss dependent on other losses

Conversion loss in an image rejection mixer is the ratio of the output i-f power level at the real port to the input r.f. power level at a constant LO drive level. It depends on the circuit losses of the r.f. and i-f quadrature hybrids, the phase balance of voltage vectors at the input to the i-f quadrature hybrids and mismatch losses at all interfaces. It also is a function of the bias and how well matched the diodes are in each of the mixers.

Circuit losses are kept to a minimum by using small transmission lines. For large scale microstrip integrated circuit production, either thick film or thin film gold plated substrates are used. Thin film, using 25 mil thick alumina substrates with gold-chrome or gold nichrome sputtered film offers potentially less loss than the thick film plated substrates. The thick film techniques as used in the design of the image rejection mixer in Fig. 5, use 35 mil thick alumina. The gold plating is 0.5 mil thick and surface finish is 20 μ in.

In practice, however, the difference in circuit losses between sputtered or thick film transmission line lengths of 1 or 2 inches is negligible. These are typical line lengths in mixer designs up to X band with 10 to 20 μ in. surface finishes.

Another major factor affecting conversion loss in image rejection mixers is the amplitude and phase balance presented to the mixer diodes. Proper balance is obtained only by designing the hybrids to very tight specifications. Phase differential must be kept to $\pm 2°$ referenced to 90° over an octave band and the power level at the hybrid output balanced to ± 0.1 dB. Phase and amplitude balance of the output i-f quadrature hybrid, is not as critical as the rf quadrature hybrid because the i-f frequencies are lower. A typical 3 dB i-f hybrid, such as used in Fig. 5, has a phase difference of $\pm 1°$ over an octave bandwidth of 45 to 90 MHz and an amplitude balance of ± 0.1 dB.

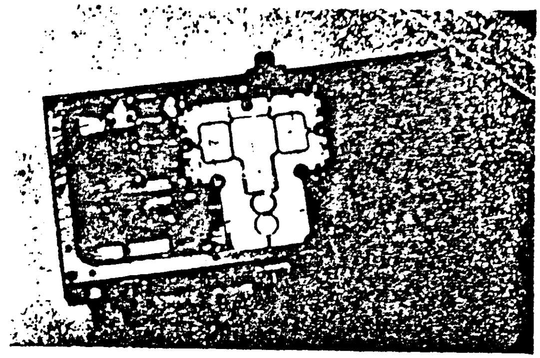
Mismatch losses are kept small by maintaining a maximum VSWR of 1.1 or better at all the interfaces between the hybrids, mixers and input/output connectors. All air gaps must be kept to a minimum and all connectors aligned so they are flush with the circuit boards. Greatest mismatch loss occurs at the diode junctions in the balanced mixer. The diodes are biased at a specified LO power level so the impedance looking into the matching network in front of the diodes is as close to 50 ohms as possible.

Conversion loss of mixer 1 and mixer 2, Fig. 2, is kept low by proper biasing and matching of the diodes. Current levels to these mixer diodes can be set at "starved" LO signal conditions typically -3 dBm to -6 dBm. At these starved LO power levels conversion loss is 1 dB to 1.5 dB higher than at LO power level of $+6$ dBm to 10 dBm. Current biasing of the diodes when set at starved LO condition can be left unchanged at high local oscillator power levels because the diodes are self biasing. More d.c. bias current will be drawn under starved LO condition. Typically, at a LO power level of -3 dBm conversion loss of 8.0 dB can be expected of an S-band image rejection mixer.

Noise figure

Noise figure of an image rejection mixer is generally equal to the sum of its conversion loss and the noise figure of the post i.f. amplifier, assuming the i.f. frequency is high enough so flicker noise is negligible. This noise figure condition also requires the image rejection mixer matched into the i.f. amplifier. Otherwise mismatch loss will have to be included in the noise figure term. In the design of the image rejection mixer, hot-carrier beam lead diodes generally are used. These unincapsulated devices simplify the network matching problem and help minimize the noise figure.

For maximum reduction in noise figure it is recommended that the i-f amplifier be integrated. Generally most low noise i-f amplifiers have a VSWR of 1.5 with a maximum VSWR of 2.0 If the real i-f port VSWR is 1.5, then a maximum cascaded VSWR of 2.25 or 3 can exist between the image rejection mixer and the i-f amplifier. This corresponds to a maximum mismatch loss of 1.25 dB.



5. S-band image rejection mixer, model 1PM 2-46Z, uses thick film techniques on alumina substrates. It operates from 2 to 4 GHz with a conversion loss of 7 dB to 8 dB, and noise figure of 8.3 to 9.6 dB over the band. Image rejection is 30 dB at 2 GHz, 44 dB at 3 GHz, and 22 dB at 4 GHz. The 2 × 2 level is 40 dB, balanced to within 3 dB at the real and image i-f ports.

Isolation depends on matching

LO-rf isolation in an image rejection mixer is the ratio of the signal present at the rf port with respect to the LO signal level at the LO port. The amount of LO radiation in most rf receiver systems is of great concern because the local oscillator frequency is often close to the receiver rf signal and thus can interfere with other receivers in the system.

Image rejection mixers are designed to provide maximum control of the LO-rf isolation and depends primarily on the isolation available in each of the mixers.

Isolation in the single balanced mixer depends on how well matched the diodes are to the 3 dB quadrature hybrid. For example in Fig. 1, the phase voltage signal level of vector, V_2 from the inphase hybrid is fed to mixer 1 and mixer 2 at ports b and b'. Due to the mismatches at the diode interface and 3 dB quadrature hybrid some of the LO signal level is reflected by the diode. The reflected signals from the diodes in each mixer appear at ports a and a' respectively. If it is assumed that the diode impedances are purely resistive, then reflected voltages are in phase at ports a and a'. In practice however, there will be some amplitude and phase difference in the reflected signal from the mixers due to parasitics and impedance mismatches in the diodes. Reflected lo signals at ports a and a' equally divide in the input r.f. 3 dB quadrature hybrid giving a further 3 dB isolation. Therefore, theoretically, isolation for lo port to rf port is equal to isolation in one mixer plus 3 dB.

Since conversion loss and noise figure are of great importance in the design of the image rejection mixer, diode impedance matching networks must be optimized for these conditions. As a result, lo-rf isolation is decreased somewhat. In general, a compromise must be made between noise figure and the lo-rf isolation—10 dB isolation is typical in single balanced image rejection mixers for frequencies up to X-band when diodes are biased for optimum conversion loss if conversion loss is degraded by 0.5 to 1 dB. Lo-rf isolation up to 13 dB is also achieveable if isolation can be further improved by inserting octave band isolators or peripheral mode isolators. Best results are achieved if the isolators are integrated into the substrates in series with the input rf signal. As mentioned before, conversion loss will be increased by the insertion loss introduced by the isolators. Obviously there is a trade off between lo-rf isolation and conversion loss and it is left to the system designers for the final decision.

In airborne ECM systems, size, weight, and ruggedness to temperature, altitude, and humidity are of considerable concern. Both microstrip and stripline designs offer better reliability to an airborne environment than individually connected components because the mixers and hybrids can be integrated on a single substrate. Microstrip offers more temperature stability than stripline, because the substrate is alumina which has a lower coefficient of expansion than copper clad dielectric boards used in stripline. Assembly and repair of diode chips is also more convenient with microstrip due to the single layer etched circuit. Microstrip offers maximum reduction of size because of its dielectric constant, $\epsilon_r = 9.7$ vs 2.5 for stripline.

Since microstrip operates in a quasi TEM mode, at frequencies above X-band it is subject to the propagation of higher order modes. Any discontinuities along the line cause the propagation of these modes. To suppress them, short circuit stubs must be placed conveniently along the transmission line. In practice, it is difficult to predict the kind of higher order modes that exist. These problems are not as severe in stripline but can occur if air gaps or improper grounding exists.

In microstrip, problems also arise due to different velocities of propagation, of a signal on the top and in the alumina substrate. This effect reduces the directivity of inphase and quadrature hybrids, particularly at X-band and above. This means stripline or micro-guide[1] techniques must be employed.

How do doubly balanced and singly balanced image rejection mixers compare in regards to these parameters? For image rejection about 20 dB up to X-band can be realized for either type. Image rejection depends on how well matched the hybrids and diodes are in the mixer. Since a double balanced has twice as many hybrid and diodes, the problem is more complicated but equal performance can be achieved.

A double balanced design offers more isolation up to 10 dB more than a single balanced image rejection mixer. This results because the rf port is isolated by an additional hybrid and a ring diode circuit.

For conversion loss, comparable results can be achieved for either type mixer, typically 8 to 9 dB at X-band and 6 to 7 dB at L-band. These conversion losses occur when the LO power level is +6 to 10 dBm. A single balanced image rejection mixer can also operate at starved LO conditions, − 3 dBm to −6 dBm, when dc bias is simultaneously applied. A double balanced design cannot and requires a higher LO level to bias the diode since a dc bias cannot be conveniently applied. Most ECM systems, for reason of size, weight and dc power regulation, prefer a starved LO condition. Also a single balanced image rejection mixer can provide the comparable conversion loss whether a 6 dBm or a +10 dBm LO signal is applied without varying the dc bias.

Both mixers offer comparable rf signal dynamic range performance. Double balanced mixers offer better spurious rejection by about 20 to 30 dB because they cancel both the even and odd products of the LO and rf signal. A single balanced image rejection mixer only cancels the even. The 2 × 2 spurious are a unique and comparable problem to both since they fall within the real i-f band.

The VSWR at all ports of a double balanced image rejection mixer is generally worse than a single balanced design because impedance matching networks cannot be easily incorporated to match the four diodes. Noise figure is the same in either design when Schottky barrier diodes are used.

Reference

1. E. G. Cristal, A. F. Podell, D. Parker, "Microguide" A New Microwave Integrated Circuit Transmission Line," 1972 IEEE-GMTT International Symposium Digest of Papers, pp. 212-214.

Acknowledgements

The author is grateful to Dr. Al Grayzel who consulted on the mathematical analysis of the diodes.

Appendix

To understand the operation of the image rejection mixer using balanced mixers, the following large signal analysis describes how each diode performs. This identifies the rf diode impedance characteristics and shows the i-f current relationship to the rf and LO signal level. The general diode equations can be applied to each diode in the double balanced mixer but the mathematics gets further complicated, necessitating a complete analysis. Fig. A-1 through A-4 shows the voltage vectors and phase relationships at the various ports with increasing detail. It is assumed that the voltage vector of the rf signal generator leads that of the LO signal generator by θ_E.

The general diode equation is given by:

$$i = I_s[e^{\beta V_d} - 1] \qquad (1)$$

where q = electron charge
K = Boltzman Constant
T = Temperature − °K
and $\beta = q/KT$

V_d, the voltage across the diode Fig. A-4 may be expressed as:

$$V_d = V_B + V_o \cos \omega_o t + V_s \cos(\omega_s t + \theta) \qquad (2)$$

where $V_o \cos \omega_o t$ = voltage across the diode due to the LO generator.
$V_s \cos(\omega_s t + \theta)$ = voltage across the diode due to the rf generator.
V_B = d.c. bias voltage developed across the diode, resulting from either an internal or external bias voltage.

Substituting the value of V_d in equation (2) in the general diode equation (1).

$$i = I_s [e^{\beta(V_B + V_o \cos \omega_o t + V_s \cos(\omega_s t + \theta))} - 1] \qquad (3)$$

The transconductance of the mixer diode, g, is by definition:

$$g = \left.\frac{dI}{dV}\right|_{v=o} \qquad (4)$$

When $V_s = 0$

$$g_{LO} = \beta I_s e^{\beta V_B} e^{\beta V_o \cos \omega_o t}$$

$$= g_o + 2\sum_{n=1}^{n=\infty} g_n \cos n\,\omega_o t \qquad (5)$$

When $V_o = 0$

$$g_{rf} = \beta I_s e^{\beta V_b} e^{\beta V_s \cos(\omega_s t + \theta)}$$

$$= g_o + 2\sum_{n=1}^{n=\infty} g_n \cos n\,\omega_o t \qquad (6)$$

where

$g_n = \beta I_s e^{\beta V_b} I_n(\beta V)$

and I_n = a modified Bessel Function.

By substituting Eq. (5) and (6) into Eq. (3)

$$i = \frac{g_{LO}}{\beta} \frac{g_{rf}}{\beta I_s e^{\beta V_b}} - I_s \qquad (8)$$

Since the mixers are used at low r-f signal levels of −10 dBm to −100 dBm and LO signal levels are much higher than rf signal levels, then g_{rf} can be approximated as follows:

$$\frac{g_{rf}}{\beta I_s e^{\beta V_s}} = I_o(\beta V_s) + 2 I_1(\beta V_s)\cos(\omega_s t + \theta) + 2 I_s(\beta V_s)\cos(2\omega_s + 2\theta)$$

Substituting $x = \beta V_s$

where $x \rightarrow 0$

$$I_o(x) \rightarrow 1 \qquad I_1(x) \rightarrow \frac{x}{2} \qquad I_2(x) \rightarrow \frac{x^2}{2}$$

Results in:

$$\frac{g_{rf}}{\beta I_s e^{\beta V_s}} = [1 + \beta V_s \cos(\omega_s t + \theta) + \beta_2^2 V_s^2 \cos(2\omega_s t + 2\theta) + \ldots\ldots] \qquad (9)$$

Similarly

$$g_{LO} = \frac{g_o}{\beta} + \frac{2 g_1}{\beta}\cos \omega_o t + \frac{2 g_2}{\beta}\cos 2\omega_o t + \ldots \qquad (10)$$

Substituting Eqs. (9) and (10) into (8) gives:

$$i = \left[\frac{g_o}{\beta} + \frac{2 g_1}{\beta}\cos \omega_o t + \frac{2 g_2}{\beta}\cos 2\omega_o t + g_o V_s \cos(\omega_o t + \theta) + 2 g_1 V_s \cos \omega_s t \cos(\omega_s t + \theta) + 2 g_2 \beta V_s^2 \cos 2\omega_o t \cos(2\omega_s t + 2\theta) \ldots - I_s\right] \qquad (11)$$

(Appendix continued on p. 76)

Appendix (continued)

Those current terms of significance in Eq. (11) are at the upper and lower sideband frequencies plus those due to the 2 x 2 products.

$$i = g_1 V_s [\cos(\omega_o t + \omega_s t + \theta) + \cos(\omega_o t - \omega_s t - \theta)] + g_2 \beta V_s^2 [\cos(2\omega_o t + 2\omega_s t + 2\theta) + \cos(2\omega_o t - 2\omega_s t - 2\theta)] \quad (12)$$

I-f current terms are:

$$i_{i\text{-}f} = g_1 V_s \cos[\omega_o t - \omega_s t - \theta] \quad (13)$$

From Eq. (11) LO current in the diode at the fundamental frequency is given by:

$$i_{Lo}(\omega_o t) = \frac{2 g_1}{\beta} \cos \omega_o t \quad (14)$$

Since the LO power level is generally higher than the rf current levels it can be assumed that the fundamental circulating current is due to the local oscillator. The equivalent ac diode impedance resulting from the local oscillator signal input is by definition:

$$R_{ac}\ (\text{diode}) = \frac{v_o(\omega_o t)}{i_{Lo}(\omega_o t)}$$

where $v_o(\omega_o t) = V_o \cos \omega_o t$

Substituting $i_{Lo}(\omega_o t)$ from eq. (14) gives:

$$R_{ac} = \frac{\beta V_o}{2 g_1} \quad (15)$$

It is now possible to define V_s and V_o in terms of V_{LO} and V_G, referred to in Fig. A-4

$$v_{LO} = \frac{2 g_1}{\beta} R_o \cos \omega_o t + V_o \cos \omega_o t$$

$$v_{LO} = \left(\frac{2 g_1}{\beta} + V_o\right) \cos \omega_o t$$

But $v_{LO} = V_{LO} \cos \omega_o t$

$$\text{Therefore } V_o = \left(V_{LO} - \frac{2 g_1 R_o}{\beta}\right) \quad (16)$$

Similarly

$$i_{rf}(\omega_s t) = g_o V_s \cos(\omega_s t + \theta)$$

$$v_G = R_o g_o V_s \cos(\omega_s t + \theta) + V_s \cos(\omega_s t + \theta)$$

$$v_G = (1 + g_o R_o) V_s \cos(\omega_s t + \theta)$$

$$v_G = V_G \cos(\omega_s t + \theta_g)$$

$$\text{Therefore } V_s = \frac{V_G}{(1 + g_o R_o)} \quad (17)$$

where $\theta = \theta_g$

Substituting Eq. (17) into Eq. (13):

$$i_{i\text{-}f} = \frac{g_1 V_G}{(1 + g_o R_o)} \cos[(\omega_o - \omega_s) t - \theta_g] \quad (18)$$

$$i_{if} = L_c \,|\, |V_g| \cos[(\omega_o - \omega_s) t - \theta_g] \quad (19)$$

where L_c is the conversion loss factor of the mixer diode.

From Eq. 12, it can be seen that current due to the 2 x 2 product of the LO and rf signals can exist within the i-f band.

$$i_{2x2} = g_2 \beta (V_G)^2/(1 + g_o R_o)^2 \cos[2\omega_o - \omega_s t - 2\theta]$$

$$= L(V_g|^2 \cos[2(\omega_o - \omega_s) t - 2\theta] \quad (20)$$

where L is the conversion loss factor for the 2 x 2 currents.

By applying Eq. (19) to each diode, as shown in Fig. 1-A, with the rf input $= K_1 V_G \angle \theta_g$ and the local oscillator signal $= K_2 V_{LO} \angle \theta_o$, then the thevenin equivalent circuit of mixer 1 can be represented as shown in Fig. A-3.

For diode D1:

$$i_1(\text{i-f}) = K L_c |V_g| \cos[(\omega_o - \omega_s) t - 90^\circ - \theta_g]$$

$$= K L_c |V_g| \sin[(\omega_o - \omega_s) t - \theta_g] \quad (21)$$

For diode D2:

$$i_2(\text{i-f}) = -K L_c |V_g| \cos[(\omega_o - \omega_s) t + 90^\circ - \theta_g]$$

$$= K L_c |V_g| \sin[(\omega_o - \omega_s) t - \theta_g]$$

Hence the total current in the i-f load in mixer 1 is given by:

$$i_{mix\,1} = i_1(\text{i-f}) + i_2(\text{i-f})$$

$$= 2 K L_c |V_g| \sin[(\omega_o - \omega_s) t - \theta_g] \quad (23)$$

By applying Eq. (19) to mixer 2 in Fig. 2, where the r.f. input signal is $K_1 V_g \angle \theta_g - 90^\circ$ and the LO signal is $V_{LO} \angle 0^\circ$, results in:

For diode D_3

$$i_3 = -K L_c |V_g| \cos[(\omega_o - \omega_s) t - \theta_g]$$

For diode D_4

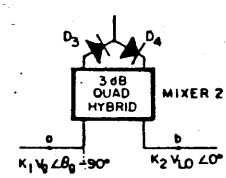
$$i_4 = +K L_c |V_g| \cos[(\omega_o - \omega_s) t - \theta_g]$$

The total current in the i-f load in mixer 2 is:

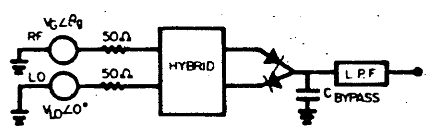
$$i_{mix\,2} = i_3(\text{i-f}) + i_4(\text{i-f})$$

$$= 2 K L_c |V_g| \cos[(\omega_o - \omega_s) t - \theta_g] \quad (24)$$

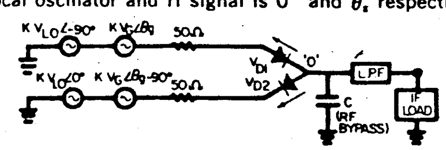
Eqs. 23 and 24 show the currents in phase quadrature. By connecting each mixer with a quadrature i-f hybrid, the currents add and subtract at the real and image ports, respectively. Ideally zero voltage exists at the image port. Eq. 20 can be further applied to other diodes and it can be shown that 2 x 2 current theoretically cancels in each mixer. This analysis can be further applied to higher order currents in the diodes.



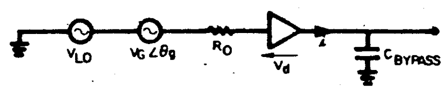
A-1. Phase relationships for mixers 1 and 2 of Fig. 1, are broken out here. The image rejection mixer uses two single balanced mixers but double balanced mixers can be used as well.



A-2. Model of a single balanced mixer assumes phase of local oscillator and rf signal is 0° and θ_g respectively.



A-3. Thevenin equivalent circuit shows further breakdown of balanced mixer in Fig. A-2 with diode potentials.



A-4. Thevenin equivalent circuit for one diode shows diodes voltage that exist in each of the mixers. A single balanced mixer has 2 such diode while a double balance mixer incorporates 4 such diodes.